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INVERSE DESIGN IN THERMAL RADIATION PROBLEMS

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ABSTRACT

Radiative transfer among surfaces is governed by integral equations. Design of practical systems where radiant heaters provide specified temperatures and heat fluxes onto a product result in inverse problems, because an outcome (the desired output of the systems) is prescribed, and the necessary design components (geometry, heater placement, heater power distribution) need to be solved for. These radiative design problems differ from other inverse problems in two important ways; their solution involves solving ill-posed integral equations, and the existence of multiple acceptable solutions is a desired outcome, instead of a nuisance. These problems can be solved using regularization and optimization methods, which are discussed and demonstrated for various design problems. Highly nonlinear problems with conduction and/or convection in addition to radiation are also described.

NOMENCLATURE

$\mathbf{A}$	matrix of coefficients
A	area
$\boldsymbol{b}$	vector of known coefficients
$\mathbf{B}$	approximate Hessian matrix
$c_j(\Phi)$	design constraints
E	emissive power, $\varepsilon\sigma T^4$
$F(\Phi)$	objective function to be minimized
$\tilde{F}$	exchange factor
$\mathbf{L}$	derivative operator, Eq. (3)
N	number of points or increments
$\boldsymbol{p}$	search direction
p	number of retained singular values
q	energy flux
$\boldsymbol{r}$	residual vector
$\mathbf{S}$	matrix of singular values
T	absolute temperature
t	time
u	location along enclosure boundary
$\mathbf{U}, \mathbf{V}$	orthogonal matrices
$\boldsymbol{x}$	solution vector
x	location along design surface

Greek symbols

α	Tikhonov regularization parameter; optimization step size;
δ	thickness of surface element
ε	surface emissivity
$\mathbf{\Phi}$	set of design parameters
ρ	density
σ	Stefan-Boltzmann constant; $5.67\times10^{-8}\ W/m^2K$

Superscripts and subscripts

b	blackbody
$cond$	conduction
$conv$	convection
DS	design surface
HS	heater surface
i , k	surface element index
k	iteration number
r	radiation
s	at radiative source (heaters)
n	time increment or unknown index

INTRODUCTION

The original need for inverse solutions in engineering heat transfer arose because experimental observations of temperature or heat flux were sometimes not available at the physical location where they were needed. These quantities instead had to be inferred from measurements at more accessible locations. Solving such problems is difficult, because the set of governing equations tend to be mathematically ill-conditioned, and predicting conditions on the remote boundary can result in multiple solutions, physically unrealistic solutions, or solutions that oscillate in space and time. Various methods may be applied for overcoming the ill-posed nature of the governing equations. For problems dominated by conduction, there are texts and monographs available that demonstrate many of these methods [1-4]. Tikhonov [5] is often credited with developing the first systematic treatment for these types of inverse problems.

Another important class of inverse problems arises in the *design and control* of thermal systems. In these problems, the designer specifies the desired output of the thermal system being designed; in most cases, this is a desired temperature and heat flux distribution over a product located on a *design surface.* The designer must then solve for the necessary inputs to the thermal system that produces the desired distributions over the design surface. The unknown conditions may be the required steady or transient temperature and energy input distributions to heaters or burners, heater or burner locations, or oven or furnace geometry.

For thermal systems dominated by radiative transfer, the problem is complicated because the thermal input at any location on the design surface may be affected by all radiant sources in the system. Thus, the mathematical form of the inverse solution is a set of integral equations that must be solved simultaneously. Some of these are Fredholm integral equations of the first kind, which are notoriously ill-conditioned. In addition, the number of unknowns in the design may be less than, equal to, or greater than the number of equations that describe the system. This implies that design and control of distributed radiative sources may be difficult, especially in problems where a transient temperature and heat flux distribution is prescribed over the design surface. The traditional design strategy has been to use trial-and-error solutions with guidance by experience to attain a viable solution.

Recent review papers discuss inverse methods for design of radiative transfer systems [6-9], and provide comprehensive reviews of research by many contributors to the field. Here we review ongoing research on thermal design and control in systems with predominant radiation using both inverse analysis and optimization methods. This paper is an update of the reviews in [6-9], and concentrates on research since the year 2000.

Inverse Design versus Data Analysis

Although the analytical techniques used for inverse design and control of radiative systems are quite similar to those used in inverse data analysis, property determination, and remote measurement, there are some significant differences in how the techniques are implemented.

First, design problems may allow significantly wider tolerances in specification of acceptable results. In such cases, *solving the inverse problem may produce multiple solutions that fall within the allowable tolerancess.* For example, Fig. 1 shows a two-dimensional enclosure. The designer may prescribe a required radiative heat flux $q(x_2, t)$ and temperature distribution $T(x_2, t)$ for the design surface (2), and seek to find the energy input distributions required on the heater surfaces (5,6,7) that will provide the desired result. As the allowable tolerances on $q(x_2,t)$ and $T(x_2,t)$ are relaxed, multiple allowable solutions for $q(x_5,t)$, $q(x_6, t)$, and $q(x_7, t)$ may appear.

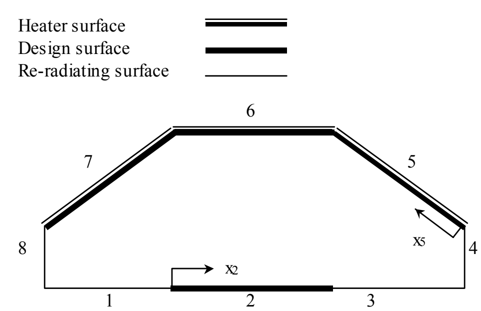

Fig. 1: Radiant furnace with design surface 2 [prescribed $T(x_2,t)$ and $q(x_2,t)$], heater surfaces 5-7 with unknown distributed power inputs $q(x_5,t)$ and unknown temperature $T(x_5,t)$, adiabatic surfaces 1, 3, 4, 8.

Multiple solutions are not acceptable in many data analyses, property determination or remote sensing problems where the thermal input or property value that provides a measured signal is sought. For the designer, however, multiple acceptable solutions are desirable, as they allow the designer to choose among the solutions based on considerations such as smoothness and ease of implementation.

Secondly, it is possible in design problems to specify conditions on the design surface for which *no acceptable physical solution may exist.* The designer may specify design surface characteristics of $q(x_2,t)$ and $T(x_2,t)$ that cannot be obtained (at least within acceptable error limits around the desired distributions) by any possible distribution of heater settings. Just because the designer wants a particular outcome, there is no *a priori* guarantee that it can be obtained! This is in contrast to data analysis problems, where in most cases a feasible solution to the inverse problem is known to exist.

Finally, designing radiant systems involves inverse solution of integral equations rather than the differential equations that arise in applications involving other heat transfer modes. Furthermore, since there are few heat transfer problems in which radiation is present but conduction and convection can be completely neglected, the governing equations are often highly nonlinear integral-differential equations.

INVERSE DESIGN METHODS

For a general transient system, the discretized energy equation for an element on the design surface (where the desired T and q are specified) can be written in terms of the emissive power $E = \varepsilon\sigma T^4$ of surface elements elsewhere in the enclosure, where ε is the surface emissivity and σ is the Stefan-Boltzmann constant. Assuming the elements are numbered so that the first N_{DS} elements are on the design surface, at any time t,

$$\sum_{j=N_{DS}+1}^{N} E_j\left(t\right) A_j \tilde{F}_{j\to i} = \rho_i c_{pi} \delta_i A_i \frac{dT_i\left(t\right)}{dt} + E_i\left(t\right) A_i - \left[q_{cond,i}\left(t\right) + q_{conv,i}\left(t\right)\right] A_i - \sum_{j=1}^{N_{DS}} E_j\left(t\right) A_j \tilde{F}_{j\to i}. \quad (1)$$

(The effects of a participating medium between the surfaces are neglected.) For a system in steady state, the transient term becomes zero; if no conduction or convection are present, then q_{cond} and q_{conv} are also zero, and the equation then becomes linear in E. Equation (1) is in terms of exchange factors from surface element k to j, $\tilde{F}_{j\to i}$. These represent the rate of energy emitted by element k that is absorbed by element j, taking into account intermediate reflection paths. Exchange factors can be used for problems with participating media and/or surfaces with directionally-dependant optical properies (such as mirrors). The exchange factors for the system usually need to be be calculated only once unless the enclosure geometry or surface radiative properties change.

All the terms on the right hand side of Eq. (1) are for the design surface elements and are therefore known; the terms on the left hand side are for the heaters or other energy sources and are to be determined. Equation (1) is thus a discretized version of an ill-posed Fredholm integral equation of the first kind. We have used various techniques for solving design problems of this type that can be grouped under the broad headings of *regularization* and *optimization*. The regularization techniques consist of modifying the governing relations to reduce their “ill-posedness”, accepting some loss of accuracy to gain a useful solution. Optimization techniques, on the other hand, approach the design problem by casting the governing relations in the conventional form with one boundary condition on the design surface fixed, and an assumed condition (most often a heat flux distribution) on the heater surface. The assumed condition is varied in a systematic way until the second boundary condition on the design surface is satisfied within acceptable limits.

A comparison of the results of regularization and optimization of a basic inverse design problem for a radiating system is presented in [9].

Regularization

A direct or explicit solution to the inverse design problem requires use of an inverse formulation. Inverse design problems are inherently ill-posed, and the corresponding discretized set of equations is ill-conditioned. Ordinary techniques (e.g., Gauss-Seidel, Gauss elimination or LU decomposition) are likely to either identify non-physical solutions with high amplitude fluctuations and/or complex absolute temperatures, or completely fail to find a solution.

To achieve an accurate and reasonable solution, the explicit system must instead be *regularized* by modifying the ill-conditioned system of equations. The solution is then subject to some error and the level of regularization must be selected so that the accuracy of the solution satisfies the designer’s needs.

Before considering regularization techniques, the ill-conditioned behavior of the system can be diagnosed by carrying out a singular value decomposition (SVD) [10,11]. The SVD of an arbitrary MxN matrix $\mathbf{A}$ is $\mathbf{A} = \mathbf{USV}^T$, where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices and $\mathbf{S}$ is a diagonal matrix of singular values, so that $S_{1,1} > S_{2,2} > \ldots > S_{N,N} \geq 0$. The inverse of $\mathbf{A}$ is then given by $\mathbf{A}^{-1} = \mathbf{VS'U}^T$, where $S'_{i,i} = 1/S_{i,i}$. If the condition number of this matrix ($S_{1,1}/S_{N,N}$) is large, small singular values dominate the inverse matrix and the solution becomes unstable. If **A** is rank-deficient, some of the singular values equal zero and the inversion process fails completely.

The most popular regularization techniques are *truncated singular value decomposition* (TSVD), the *conjugate gradient method* (CGM) and *Tikhonov regularization* (TR).

Truncated Singular Value Decompositoon is based on a singular value decomposition of **A**. The solution uses the pseudo-inverse matrix that is formed by filtering or truncating small singular values; the solution using the p largest singular values becomes

$$x_n = \sum_{k=1}^{p} V_{n,k} \frac{b_m U_{m,k}}{S_{k,k}}, \quad n = 1,2,\ldots,N, \qquad (2)$$

where p has a value less than or equal to the rank of **A** [11]. Retaining different numbers of singular values yields alternative solutions. Those with acceptable accuracy present possible alternatives to the designer.

A plot of the variation of the residual norm against the solution norm is often called the "L-curve", and can be used to select an appropriate solution. The residual norm represents the solution accuracy while the solution norm represents the smoothness of the solution. The L-curve usually has a corner where further regularization significantly increases the norm of the residual with little improvement in solution smoothness. The optimal or desired solution often lies in the vicintiy of the corner [11].

The CGM is based on the conjugate gradient minimization of the functional $F(\mathbf{x}) = [\mathbf{A}\ (\mathbf{x}_{\mathbf{exact}}\text{-}\mathbf{x})]\cdot(\mathbf{x}_{\mathbf{exact}}\text{-}\mathbf{x})$, which is minimized when $\|\mathbf{A}\boldsymbol{x} - \boldsymbol{b}\| = 0$. For inverse problems, each minimization step corresponds to a unique solution having distinct accuracy and smoothness characteristics. These solutions can then be used to construct an L-curve to select the optimal solution. Small memory requirements, computation economy, robust convergence characteristics, and the ability to store and use the original matrix makes CGM the method of choice for large systems [4, 12].

Like CGM, Tikhonov regularization is based on minimizing a functional. In this method, the functional is equal to the L_2 norm of the residual vector plus an added shape constraint,

$$F(\boldsymbol{x}) = \|\mathbf{A}\,\boldsymbol{x} - \boldsymbol{b}\|_2 + \sum_{i=1}^{p} \alpha_i^2 \left[\|\mathbf{L}_i(\boldsymbol{x} - \boldsymbol{x}_0)\|\right]^2. \qquad (3)$$

For a p^{th} order scheme, $\mathbf{L}_i$ approximates the discretized i-th derivative operator and α_i is the i^{th} order regularization parameter. Using a small regularization parameter results in an accurate solution by emphasizing minimization of the residual norm. Using a large regularization parameter, on the other hand, results in a solution with improved smoothness characteristics. For the standard or zeroth order TR scheme, $p = 0$ and $\mathbf{L}_i$ becomes the identity matrix $\mathbf{I}$, leading to the set of linear equations $(\mathbf{A}^T\mathbf{A}+\alpha_0^2\mathbf{I})\ \boldsymbol{x} = \mathbf{A}^T\boldsymbol{b} + \alpha_0^2\mathbf{I}\boldsymbol{x}_0$. Using the correct regularization parameter results in an optimal solution that is both smooth and sufficiently accurate for the designer's needs.

Enclosure Design Using Regularization

Inverse boundary condition design can be applied to steady and transient applications. Combined mode problems are treated by TSVD in [13], and a combined conduction-radiation design problem is solved in [14]. A more complex geometry with the presence of an absorbing-emitting media is presented in [15]. Erturk et al. [16] compare three regularized solution techniques (TSVD, CGM and Bi-CGM) for a three-dimensional enclosure problem with an absorbing emitting and anisotropically scattering medium. All of these studies consider steady radiating systems.

The inverse formulation is also applicable for transient systems. Erturk et al. [17-19] considers the design of an enclosure for material processing. In each case, the goal is to heat the design object following a specified temperature history while keeping it spatially isothermal. The inputs required for the heaters are determined using the CGM at every time step using a constant regularization level to prevent instabilities along time. The object can be heated in a batch process [17] or during passage through a radiant oven [18,19].

For example, the objective of the design problem shown in Fig. 2 is to maintain a uniform temperature over the design surface as it is heated according to a prescribed heating curve which is a cubic function of time. The transient heater settings that satisfy this requirement are found using the same number of CJ regularization steps at each time step. As shown in Fig. 3, substituting these heater settings into the forward problem results in a near-uniform design surface temperature that closely follows the desired temperature throughout the process.

In [20], TSVD was used to solve a similar problem. It was found that saving 6 singular values from a system of 65 independent equations with 130 unknowns provided a temperature distribution that was within one percent of the desired value.

If the heater inputs found by the inverse solution are applied in a physical system, there will be some difference between the measured and desired conditions on the design surface because the heat transfer models used for the inverse solution are based on assumptions and approximations; furthermore, the radiative data is often of questionable accuracy. Thus, a control algorithm would need to be used to correct the heater inputs for this modelling error. The use of an artificial neural net (ANN) based control algorithm to account for modelling error is demonstrated in [18].

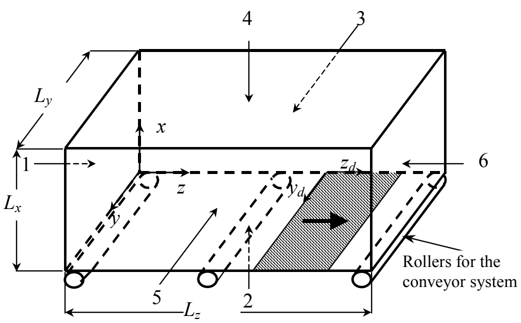

Fig. 2: Radiant Heating by Heaters on Surface 4 of Design Surface in Roll-Through Furnace [19].

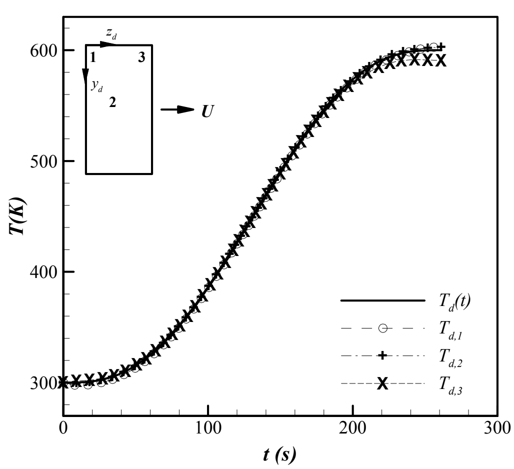

Fig. 3: Calculated Temperatures at Selected Locations on the Design Surface as Computed from Heater Distributions found by Conjugate Gradient Solution.

Experimental Validation

A design validation study was done for a steady state problem with dominant radiative heat exchange [21]. The experimental apparatus is a radiatively heated test chamber designed to reproduce the main characteristics of a silicon wafer rapid thermal processing (RTP) chamber, developed by the University of Texas, NIST, the SensArray Corporation and International SEMATECH [22, 23] for calibration of commercial temperature sensors. The apparatus is an axisymmetric radiatively heated chamber composed of a cylindrical stainless steel, a water cooled vacuum vessel that contains the design surface (in this case, the silicon wafer to be processed) and the heating elements, composed of three independently powered concentric zones, plus radiation shields placed on the bottom, top and sides. The system was instrumented to map temperature fields on the wafer and radiation shields.

The goal of the validation study is to predict the power input to the innermost and outermost heating zones (the middle zone is rarely used in the particular experimental apparatus) needed to keep the silicon wafer at a uniform temperature. The radiative heat transfer was modeled using the Monte Carlo method to handle the complicated geometries, shading effects and spectral and directional dependencies of the optical properties. Optical properties (reflectivity and specular/diffuse ratio) were measured for the molybdenum radiation shields and silicon wafer, while standard optical properties were assumed for other surfaces. The optimal heater settings were calculated using the conjugate gradient regularization method.

The optimal heater input solution for a desired uniform wafer temperature of 873 K was imposed on the experimental apparatus and resulted in a wafer temperature uniformity of ± 6.3 K. The experimental error is ± 1.0 K. For a second case, a desired temperature of 1073 K was imposed on the wafer; using these results to set the heater power output resulted a wafer temperature of about 1039.9 K and a uniformity of ± 2.9 K. In both cases, the wafer temperature was predicted within 3%, and fair uniformity was achieved.

An additional experimental validation of inverse analysis using TSVD is presented in [24], where a set of high-intensity visible light sources is controlled based on the predicted energy inputs required to produce a desired distribution of incident light on a set of detectors.

Optimization

Like conventional trial-and-error design, optimization techniques use an iterative process to arrive at the final design configuration. The performance of a particular design configuration is evaluated at each iteration. If it does not satisfy the design requirements, the configuration is modified and checked again. This process is repeated until a satisfactory design configuration is identified.

The efficiency of this process and the quality of the final design depend on how much the design performance improves at each iteration. While the trial-and-error technique relies solely on the designer's intuition and experience to improve the design, optimization techniques modify the design configuration systematically, based on sensitivity information and numerical algorithms that maximize the improvement between successive iterations. Consequently, optimization techniques require far fewer iterations than the trial-and-error approach, and the final solution is usually near optimal.

The first step of the optimization technique is to specify a set of n design parameters contained in, $\mathbf{\Phi}$, which control the design configuration. Next, an objective function, $F(\mathbf{\Phi})$, that quantifies the merit of a particular design is defined in such a way that the minimum of $F(\mathbf{\Phi})$ corresponds to the ideal design outcome. Design constraints, $c_i(\mathbf{\Phi})$, which define the domain of the design parameters may be also be specified. Writing the problem in this way effectively transforms the design problem into a multivariate minimization problem, where the goal of the analysis is to identify the set of design parameters $\mathbf{\Phi}^*$ that satisfies

$$F(\mathbf{\Phi}^*) = Min[F(\mathbf{\Phi})], \mathbf{\Phi} \in \Re^n, \tag{4}$$

subject to

$$c_i(\mathbf{\Phi}) = 0, \quad i = 1 \ldots m',$$
$$c_i(\mathbf{\Phi}) \le 0, \quad i = m'+1 \ldots m. \tag{5}$$

Consider the enclosure design problem shown in Fig. 4, where the goal is to find the enclosure geometry and heater settings that produce a uniform heat flux of q_s^{target} over the bottom surface. The design parameters in $\mathbf{\Phi}$ specify the enclosure geometry and heat flux distribution over the heater surface. The objective function is defined as the variance of the realized heat flux distribution from the desired heat flux distribution, evaluated at N_{DS} discrete points over the design surface,

$$F(\mathbf{\Phi}) = \frac{1}{N_{DS}} \sum_{j=1}^{N_{DS}} \left[q_j(\mathbf{\Phi}) - q_j^{target} \right]^2. \tag{6}$$

Constraints are used to restrict the dimensions of the enclosure geometry and to ensure that the heat flux distribution over the heater surface remains strictly non-negative.

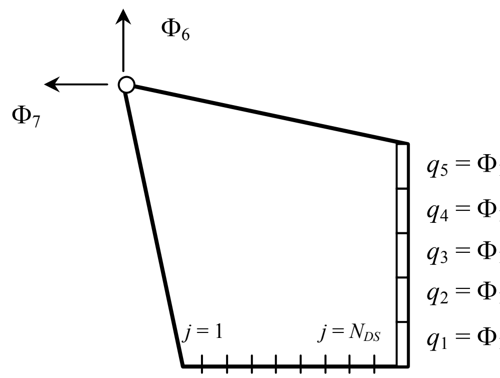

Fig. 4: Solving a radiant enclosure design problem using optimization.

Gradient-based minimization is used when $F(\mathbf{\Phi})$ is continuously differentiable, and work according to the following scheme: at the k^{th} iteration, the set of design parameters, $\mathbf{\Phi}_k$, is checked to see if it minimizes $F(\mathbf{\Phi})$, usually by seeing if $|\nabla F(\mathbf{\Phi}^k)| \approx 0$. If not, a search direction, $\boldsymbol{p}^k$, is selected based on the curvature of $F(\mathbf{\Phi})$ at $\mathbf{\Phi}^k$. Once a search direction has been chosen, a scalar step size, α^k, is found. Finally, a new set of design parameters is formed by taking a "step" in the $\boldsymbol{p}^k$ direction,

$$\mathbf{\Phi}^{k+1} = \mathbf{\Phi}^k + \alpha^k \boldsymbol{p}^k. \tag{7}$$

This process continues until $\mathbf{\Phi}^k \approx \mathbf{\Phi}^*$.

The performance of gradient-based minimization methods depends on how the search direction is chosen. One of the simplest methods is steepest descent, where $\boldsymbol{p}^k$ is set equal to the negative of the gradient,

$$\boldsymbol{p}^k = -\nabla F(\mathbf{\Phi}^k), \tag{8}$$

i.e. the direction of steepest descent. Many iterations are required to reach $\mathbf{\Phi}^*$, and accordingly this technique is rarely used.

Newton's method relies both on the first- and second-order curvature information to choose a search direction, and usually requires the fewest iterations to find Φ^*. Newton's direction is found by solving

$$\nabla^2 F\left(\Phi^k\right) \boldsymbol{p}^k = -\nabla F\left(\Phi^k\right), \tag{9}$$

where the Hessian matrix contains the second-order objective function sensitivities, i.e. $\nabla^2 F_{pq}(\Phi^k) = \partial^2 F(\Phi^k)/\partial\Phi_p\partial\Phi_q$.

Although Newton's method requires the fewest iterations to find Φ^*, the computational effort required to find $\nabla^2 F(\Phi^k)$ often makes its use impractical. The quasi-Newton method avoids calculating the second-order sensitivites by using first-order curvature information "built-up" over successive iterations to form a matrix $\mathbf{B}^k$ that approximates the Hessian matrix. The search direction is then found by solving

$$\mathbf{B}^k \boldsymbol{p}^k = -\nabla F\left(\Phi^k\right). \tag{10}$$

Quasi-Newton methods need more iterations to reach Φ than Newton's method, but often require less computational effort overall.

Enclosure Design Using Optimization

The simplest types of design problems involve diffuse-walled enclosures with transparent media. Daun et al. [27] describe an optimization methodology for finding the optimal heater settings that produce a desired temperature and heat flux distribution over a design surface. The objective function is minimized using Newton's method. Because of the form of the governing integral equations, the first- and second-order radiosity sensitivities are found efficiently by post-processing the radiosity solution; these sensitivities are then used to calculate the elements in the gradient vector and Hessian matrix at every iteration, which in turn are used to solve for $\boldsymbol{p}^k$ through Eq. (9). A three-dimensional diffuse-walled enclosure with and without a participating medium was analyzed using optimization in [28].

Optimization has also been applied to design the enclosure geometry rather than the heater energy distributions to meet desired distributions on the design surface [29-30]. A technique for optimizing the geometry of diffuse-walled radiant enclosures is described in [29]. The enclosure surfaces are represented by Bézier curves, and the design parameters are their control points. The first- and second-order objective function sensitivities are calculated by post-processing the radiosity solution, and Newton's method is used to minimize the objective function.

Geometric optimization of enclosures containing specularly-reflecting surfaces is encountered in design of solar collectors, light fixtures, and infrared heaters. Daun et al. [30] present a stochastic optimization technique that uses a Monte Carlo radiation simulation coupled with a Kiefer-Wolfowitz minimization algorithm [25, 26] to identify the enclosure geometry that produces the desired temperature and heat flux distributions over the design surface. This technique was demonstrated by solving the design problem shown in Fig. 5; the heat flux distributions over the design surface based on the initial and optimal heater settings are shown in Fig. 6, and it is seen that the optimal solution closely matches the desired solution of $q(x) = -1$ W/m^2.

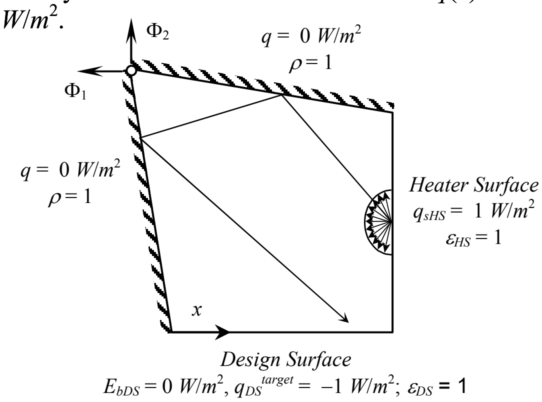

Fig. 5: Geometric optimization of an enclosure containing both specular and diffuse walls [30].

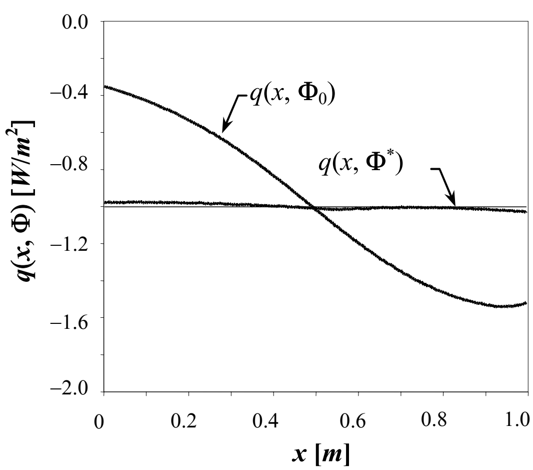

Fig. 6: Initial and optimal heat flux distributions over the design surface for the enclosure shown in Fig. 5 [30].

Optimization has also been used for the case of transient batch heating in manufacturing. This was first demonstrated by Fedorov et al. [31], who determined the optimal heater settings in a roll-through annealing furnace operating under steady-state conditions.

Daun et al. [32] recently demonstrated an optimization technique for solving fully transient problems, using design constraints to assure that input energy to the heaters is always positive. This is done by representing the heater outputs using cubis splines where the basis functions are defined using nondimensional time and the control points are design parameters. Thus, the power output of the h^{th} heater at time $\tau = t/t_{max}$ is given by

$$q_h(\Phi) = (1-\tau)^3 \Phi_h + 3(1-\tau)^2 \tau \Phi_{h+1} + 3(1-\tau)\tau^2 \Phi_{h+2} + \tau^3 \Phi_{h+3}, \tag{11}$$

where $\{\Phi_h, \Phi_{h+1}, \Phi_{h+2}, \Phi_{h+3}\}^T$ is a subspace of Φ. This reduces the dimensionality of the optimization problem, regularizes the transient power output of the heaters, and facilitates the use of boundary constraints on the heater outputs, since $l_{b,h} \leq q_h(\Phi) \leq u_{b,h}$ is enforced by applying the same bounds to the design parameters contained in the corresponding subspace of Φ.

This technique was demonstrated by solving the design problem shown in Fig. 7, with surface properties summarized in Table 1. The objective was to find transient heater settings that heat the design surface from 300 *K* to 500 *K* according to a linear ramp rate, while simultaneously maintaining a uniform spatial temperature distribution throughout the process. (The enclosure is in thermal equibrium at 300 *K* when the heating begins.)

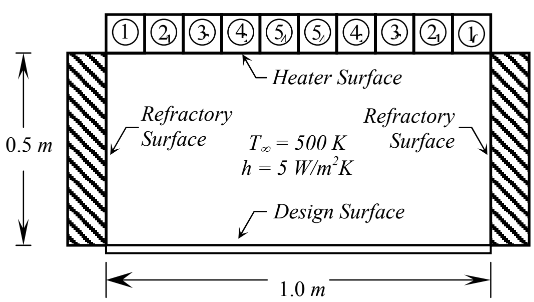

Fig. 7: Geometric optimization of transient heater settings [32].

The optimal transient heater settings are shown in Fig, 8, and are normalized by σT_s^4 where T_s = 1000 *K*. The corresponding design surface temperatures closely match the set-point temperature and are nearly uniform throughout the process.

Table 1: Surface properies of design problem shown in Fig. 7 [32]

	Heater Surface	Refractory Surface	Design Surface
κ [$W/m\ K$]	1.0	1.0	63.9
ρ [kg/m^3]	2645	2645	7832
c [$J/kg\ K$]	960	960	487
δ [m]	0.1	0.1	0.02
ε	0.8	0.8	0.4

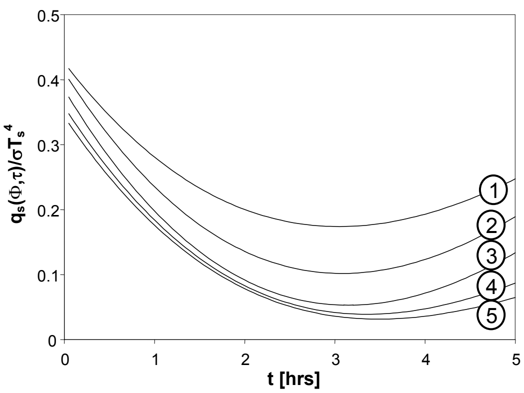

Fig. 8: Optimal transient heater settings [32].

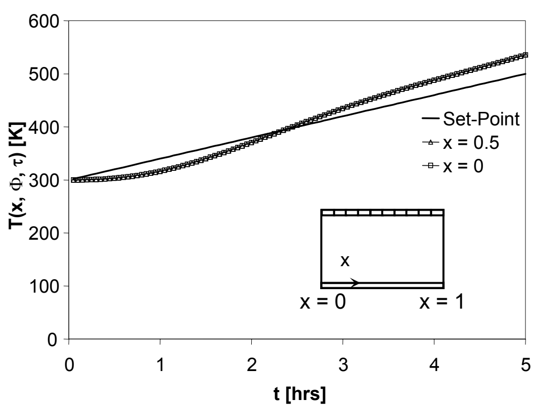

Fig. 9: Design surface temperature obtained using optimal transient heater settings [32].

Comparison of Regularization and Optimization Design Techniques

A comparison of regularization and optimization solutions for a steady-state inverse design problem in an enclosure without participating medium is presented in [9]. This study concluded that although solutions obtained through optimization require more iterations than the regularization method, optimization allows inclusion of constraints in the formulation (e.g., requiring strictly nonnegative power supply to the heaters). Thus, the solution by optimization may be more useful than the unconstrained solution from regularization, where it is difficult to *a priori* impose the constraints. A comparison from [9] is shown in Figs. 10-12.

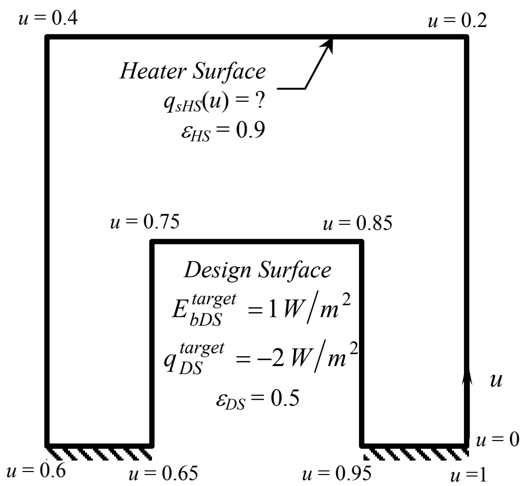

Fig. 10: Enclosure design problem used to compare regularization and optimization solutions [9].

Tikhonov regularization and Newton's method for optimization were used to determine the required heater values to produce the specified conditions over the design surface; these heater values are shown in Fig. 11. While a discrete heat flux distribution over the heater surface is easy to enforce in the optimization method, this is more difficult in the regularization method. Accordingly, the heat flux distribution is assumed to be continuously variable over the heater surface in the latter case.

As shown in Fig. 11, the Tikhonov solution involves regions over the heater surface involving negative heat flux, which is not a useful engineering solution since some of the heaters would act as heat sinks on the heater surface. In order to implement this solution, all negative heater fluxes would likely be set equal to zero at the expense of the solution quality over the design surface, as shown in Fig. 12.

In contrast, the heat flux distribution over the heater surface is forced to by nonnegative throughout the optimization process by imposing nonnegativity constraints on the design parameters through Eq. (9). As shown in Figs. 11 and 12, this results in a solution that both closely satisfies the desired conditions over the design surface, and could be easily implemented in a practical setting.

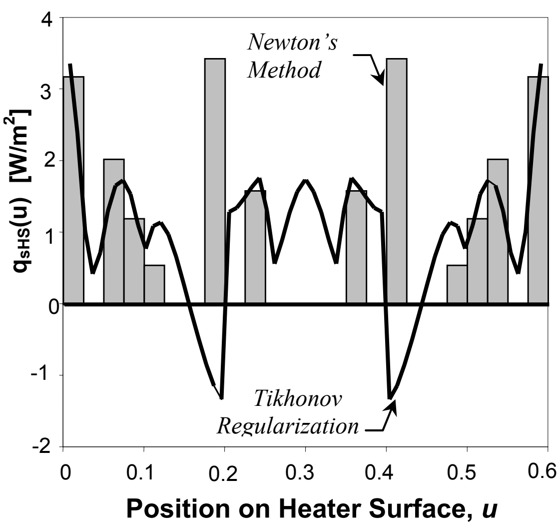

Fig. 11: Heat flux distributions over the heater surface obtained through the optimization (Newton) and inverse (Tikhonov) design methodologies [9].

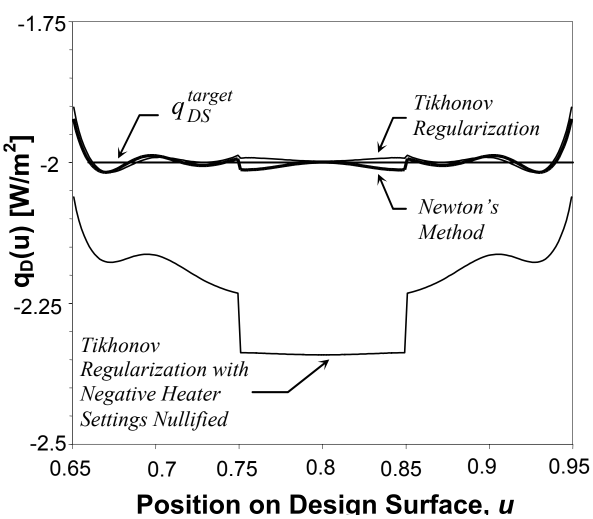

Fig. 12: Heat flux distributions over the design surface obtained through the optimization (Newton) and inverse (Tikhonov) design methodologies [9]

UNRESOLVED PROBLEMS AND FUTURE WORK

Although a considerable amount of progress has been made in developing methods for design of radiating systems, there remains significant work to be done. In specifying the conditions on the design surface, for example, the design engineer may choose conditions for which there is no physical solution, i.e. the solution might not be achievable without unacceptable heater conditions (coolers in place of heaters, or negative or imaginary absolute temperatures on the heaters). Such solutions may satisfy the conditions on the design surface mathematically, but they are not useful engineering solutions. An *a priori* determination of the existence of acceptable physical solutions would save a lot of fruitless calculation, but is not yet at hand.

These techniques could be incorporated into a useful, easy-to-use design tool by automating the optimization and inverse solution procedure. The designer would first specify a desired solution accuracy, and the algorithms outlined above would then proceed to identify an acceptable solution without user intervention by determining the best number of singular values, the best Tikhonov parameter, the correct number of conjugate gradient steps, the best level of optimization, etc. However, it remains to be determined what approach would best accommodate such designer-friendly methods.

Finally, it should be possible to design a radiant enclosure by simultaneous optimization of both the geometry and heater power distribution, rather than optimization of each factor independently. This is a subject of present research.

CONCLUDING REMARKS

The methods and applications of inverse thermal design and control have been outlined and discussed, and some of the remaining research issues have also been addressed.

The authors have observed that both regularization and optimization techniques have advantages in certain settings. Regularization, particularly TSVD, can provide insight into the reasons for the ill-posed nature of a particular problem, and can also indicate how much regularization is required in order to extract an acceptable or practical solution from an ill-posed problem. However, TSVD can be a costly process in terms of computational time and effort, and may not be practical for large systems involving a great number of equations. The conjugate gradient technique provides similar regularization to TSVD at less cost in computational time and effort, but in transient problems it can introduce unacceptable fluctuations along time unless the same low degree of regularization is maintained at each time step throughout the process.

Optimization provides an alternative approach that allows easy incorporation of constraints into the formulation, and can provide economical solution of complex problems. For problems in which the kernel of the integrals is a variable (problems in which the radiating enclosure geometry is to be optimized), optimization is the only viable solution method.

It is hoped that the thermal design community will see the advantages of these methods, and adopt them in the design of actual thermal systems where radiation plays an important or dominant role.

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